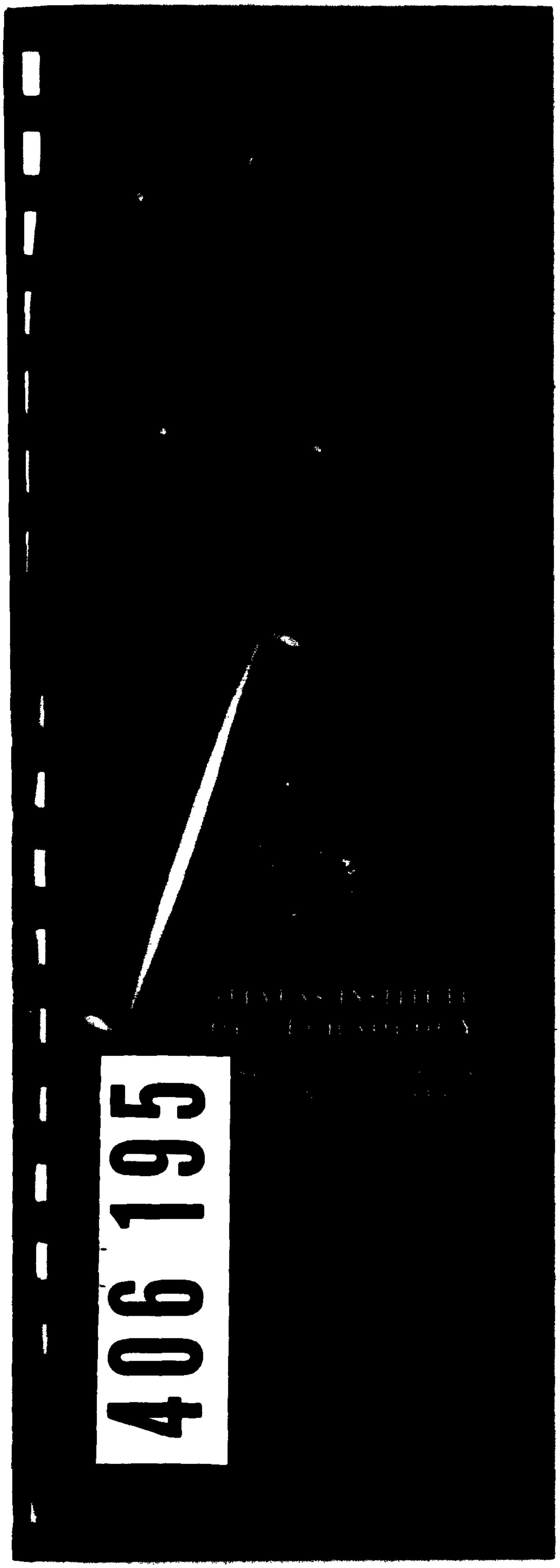

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Report 855

LONGITUDINAL BLADE-FREQUENCY FORCE INDUCED BY A PROPELLER ON A PROLATE SPHEROID

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Bureau of Ships
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ABSTRACT

An expression has been developed for the longitudinal component of the vibratory force exerted on a prolate spheroid by the operation of a marine propeller in a space-varying field (wake). Two evaluation schemes have been considered: one by integration of the pressure signal over the surface of the ellipsoid and the other by means of Lagally's theorem with the ellipsoid represented by a known source-sink distribution. Numerical calculations indicate the important role played by propeller clearance and slenderness ratio in the magnitude of the vibratory force.

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INTRODUCTION

During the past few years, theoretical investigations have been undertaken at Davidson Laboratory of Stevens Institute of Technology into the vibratory pressure and velocity field around an operating marine propeller with the object of determining the forces exerted on nearby bodies. Early studies[1,2] were restricted to the case of uniform inflow to the propeller which is represented by a line-vortex array to give the effect of loading and by a source-sink distribution for the blade thickness effect. Later studies of the propeller field treated nonuniform inflow conditions since in the case of a marine propeller located behind a hull, nonuniformity of the inflow field is a common occurrence. In these studies[3,4] which were concerned with both acoustic and hydrodynamic media, the propeller was represented by axial and tangential doublet distributions on the propeller blade axis with strength depending on radial and angular position. In considering propeller operation under nonuniform inflow conditions (wake), the analysis takes cognizance of a situation more realistic than the open water condition, since the effect of the boundary is taken implicitly into account through the wake formation. In other investigations, initiated by Davidson Laboratory toward fulfillment of the long-range objective of the series, expressions have been developed for the vibratory forces and moments produced by a marine propeller on a doubly-infinite rigid plate[5] and an infinitely long rigid strip.[6] Results of the latter study are surprisingly simple and indicate that a vibratory force of considerable magnitude could be obtained from the transient field generated by an operating propeller.

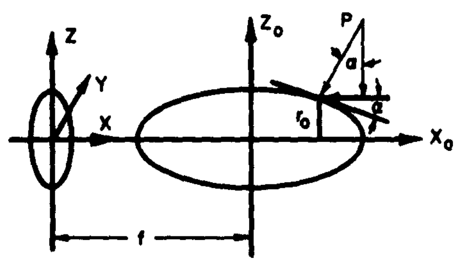
The present study endeavors to determine the vibratory forces exerted on an ellipsoid of revolution by a marine propeller operating in a specified wake behind a prolate spheroid.

The study has been carried out under Bureau of Ships Fundamental Hydrodynamics Research Program S-R009 01 01, Contract Nonr 263(16) and administered by David Taylor Model Basin.

QUASI-STEADY AND UNSTEADY VIBRATORY FORCE ON A PROLATE SPHEROID

Vibratory Force

The propeller disc is located at a distance f from the center of the spheroid whose major axis coincides with the longitudinal axis of the propeller (see sketch). The prolate spheroid is defined in terms of its major axis 2a and its eccentricity e. Two cartesian coordinate systems are used with the same direction of positive axes but with different origin. The axes located at the propeller center are designated by x, y, z and the axes with origin at the center of the ellipsoid of revolution by x_o, y_o, z_o.



Sketch No. 1

The relation between the cartesian coordinates x_o, y_o, z_o and ellipsoidal coordinates ζ, μ, ψ are given by

$$x_o = k\mu\zeta \qquad 1 < \zeta \leq \infty$$

$$y_o = k(1-\mu^2)^{1/2} \, (\zeta^2-1)^{1/2} \cos\psi \qquad -1 \leq \mu \leq 1$$

$$z_o = k(1-\mu)^{1/2} \, (\zeta^2-1)^{1/2} \sin\psi \qquad 0 \leq \psi \leq 2\pi$$

For a given ellipsoid of known eccentricity e and major semi-axis, a,

$$\zeta_o = \frac{1}{e}, \; k = ae, \; \mu = \frac{x_o}{a} = \xi \tag{1}$$

The radial distance $r_o = (x_o^2 + y_o^2)^{1/2} = a(1-e^2)^{1/2}\,(1 - \xi^2)^{1/2}$

and the x_o-cosine direction of the unit vector tangent to the surface ζ = constant is given by

$$\cos\alpha = \left(\frac{1 - \xi^2}{1 - e^2\xi^2}\right)^{1/2} \tag{2}$$

The propeller located behind a ship operates in space-varying inflow conditions so that its rotating blades experience a time-dependent gust. On the basis of this physical reasoning an approximate theory has been developed[7] utilizing two-dimensional unsteady theory in a stripwise fashion. It is true, on the other hand, that the pressure of the propeller disturbs the fluid field around the body so that the stream lines adjacent to the body are distorted. This amounts to distortion of the original shape of the body. The effect on the boundary of the presence of the propeller, the so-called "image effect" necessary to restore the boundary, was rigorously evaluated in the case of the doubly-infinite rigid plate[5] and later for an infinitely long rigid cylinder.[8,9] In the latter studies it was found independently that the vibratory force exerted on the cylinder due to the "image effect" is identically the same as that developed directly by the propeller action. To the best of the authors' knowledge, there is not at present a method of constructing the "image potential" for the case of an ellipsoid of revolution in the presence of the propeller. This case is complicated further by the fact that the propeller potential is more conveniently expressed in cylindrical coordinates whereas the fluid potential around the ellipsoid of revolution is expressed in terms of ellipsoidal coordinates. This study, therefore, while taking cognizance of the effect of the boundary on the propeller, ignores the "image effect." It is believed that for bodies of practical interest--that is, very slender bodies--the effect of the so-called "image system" will be secondary. The axial velocity induced by the action of the propeller is much smaller than the forward velocity of the body, so that the effect of the distortion in the axial direction will be small.

There are two possible schemes for evaluating the axial vibratory force exerted on the prolate spheroid: 1) by integrating the transient pressure over the surface of the body; 2) by means of Lagally's theorem since the spheroid can be represented by a line source-sink distribution of known strength. It is to be noted that for a propeller located on the body axis, only the fore-and-aft force is present; the transverse force in any direction will be nil due to the symmetry.

According to scheme 1, the axial force ΔF_x exerted on a ring of radius r_o located at x_o, will be given by

$$\Delta F_x = - P \sin \alpha \quad r_o \, d\theta \, \frac{dx_o}{\cos\alpha}$$

where P is the pressure distribution arising from the operating propeller. The resultant axial force exerted on the ellipsoid will be given therefore by

$$F_x = \int_{f-a}^{f+a} \int_0^{2\pi} P r_o \tan \alpha \, d\theta \, dx$$

or

$$F_x = a(1-e^2) \int_{-1}^{1} \int_0^{2\pi} P(r_o, \xi) \; \xi \, d\theta \, d\xi \qquad (3)$$

by utilizing eq. 2. In this form the expression for the pressure arising from the propeller is written in terms of the nondimensional coordinate ξ of the prolate spheroid. The pressure field emanating from the operating propeller is made up of a superposition of axially and tangentially directed doublets. The latter doublet distribution is in a plane normal to the x-axis and will not contribute to the axial component of the vibratory force, whereas the first one, which is designated as the thrust producing pressure, P_T, will be the only contributor in eq. 3.

In the second scheme the ellipsoid of revolution is represented by a line segment distribution of sources and sinks between the focal points, of strength[10] given by

$$\frac{U}{E} x_o$$

where U = forward velocity of the hull

$$E = \frac{2e}{1-e^2} - \ln \frac{1+e}{1-e} \qquad (4)$$

Then the interaction force between the propeller and the element of the singularity representing the ellipsoid will be given by application of Lagally's theory as

$$\Delta F_x = - 4\pi\rho(f-x) \frac{U}{E} dx \cdot u_x \Big|_{r=0} \qquad (5)$$

where

u_x = axial component of the velocity induced by the propeller at any point on the axis of the ellipsoid.

It is known, however, that the instantaneous linearized pressure in the field of an operating propeller is given by

$$P/\rho = \frac{\Omega\partial\phi}{\partial\theta} - U \frac{\partial\phi}{\partial x} \qquad (6)$$

where Ω = propeller angular velocity and ρ = fluid density and ϕ the velocity potential. The first term is the torque-associated pressure signal; the second is the pressure component associated with propeller thrust. The torque-associated pressure acting in a plane normal to the x-axis will not contribute to the x-component of the force as it integrates to zero on any circle with center on the propeller axis. In addition, its value on the x-axis is identically zero, since $r = 0$.[3] Hence eq. 6 is reduced to

$$-\frac{\partial\phi}{\partial x} = u_x = \frac{P_T}{\rho U}$$

which upon substitution into eq. 5 and integration leads to

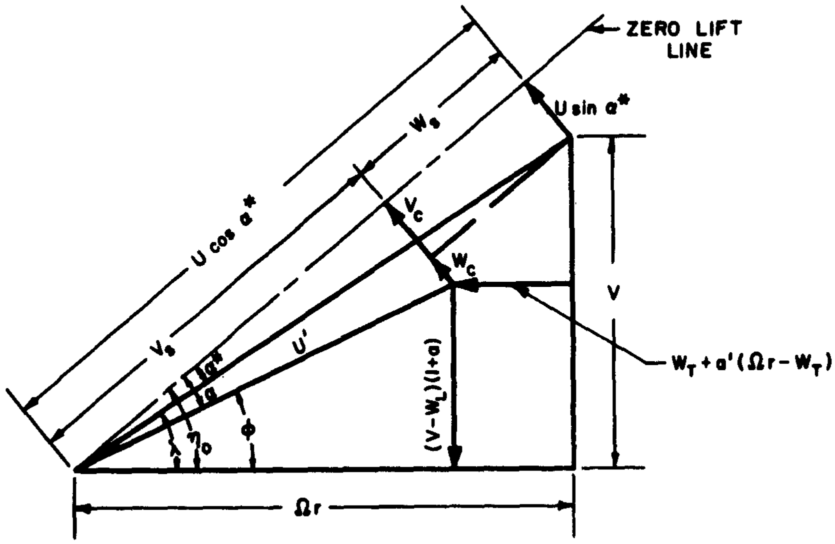
$$F_x = \frac{4\pi}{E} \int_{f-k}^{f+k} (x-f)\ P_T\Big|_{r=0}\ dx \tag{7}$$

where k = ae which defines the locus of the focal points. The axial component of the vibratory force exerted on the prolate spheroid by the propeller action is determined either by eq. 3 or eq. 7, respectively, once the pressure on the hull or on its axis is determined.

Pressure Field

The pressure field emanating from a marine propeller operating under space-varying inflow conditions has been the subject of ref. 3 and 4. The flow about propeller blades rotating through the space-varying ship wake is equivalent to that of a wing moving through a sinusoidal gust. Although the problem is of very pronounced three-dimensional character (low aspect ratio of the blades, three-dimensional gust), the two-dimensional unsteady aerodynamic theory has been utilized in a stripwise fashion as the best expedient in the present state of the art. It is assumed that a two-dimensional flow condition exists at every blade section and that three-dimensional effects can be taken into account by Burrill's[11] method which applies correction factors to the axial and tangential components of the inflow velocity relative to the propeller.

At any radial section of the blade the velocity components and the resultant velocity are shown in sketch No. 2 below:



where V	=	propeller forward velocity
Ω	=	angular propeller velocity
a, a'	=	axial and tangential correction factors, respectively[11]
U, U'	=	resultant fluid velocity with respect to the propeller in the absence and presence of the wake, respectively
W_L, W_T	=	longitudinal and tangential wake velocity, respectively
α^*	=	angle of attack
α	=	effective angle of attack
ϕ	=	hydrodynamic pitch angle
η_o	=	effective pitch angle

If the instantaneous velocity U' is resolved along and normal to the chord, then the corresponding surge and cross velocities are given as

$$V_s = U \cos \alpha^* - W_s$$

$$V_c = U \sin \alpha^* + W_c$$

where

$$W_s = \left[W_T + a' (\Omega r - W_T)\right] \cos \eta_o + \left[W_L - a(V - W_L)\right] \sin \eta_o$$

$$W_c = \left[W_L - a (V - W_L)\right] \cos \eta_o - \left[W_T + a' (\Omega r - W_T)\right] \sin \eta_o$$

For small angles, the following four combinations of surge and cross velocities will simulate the flow around the propeller and give rise to corresponding components of lift produced by a propeller operating in a wake:

	Surge Velocity	Cross-flow Velocity	Lift
1	U	$U\alpha^*$	$L^{(1)}$
2	U	W_c	$L^{(2)}$
3	$-W_s$	$U\alpha^*$	$L^{(3)}$
4	$-W_s$	W_c	$L^{(4)}$

(8)

The lift at the corresponding sections is determined by utilizing the results of unsteady aerodynamic theory in terms of the harmonic constituents of the wake flow. The elementary thrust and torque developed by a propeller section at a radial distance r_1 will be given by

$$\frac{dT}{dr_1} = \pi r_1 \ \rho \frac{\sigma}{B} C_L U'^2 \cos \lambda$$

$$\frac{dQ}{dr_1} = \pi r_1^2 \ \rho \frac{\sigma}{B} C_L U'^2 \sin \lambda \qquad (9)$$

where

C_L = lift coefficient per blade section

B = number of blades

$\sigma = \frac{bB}{2\pi r_1}$ = solidity

b = chord length at each propeller section

On the assumption of constant blade-loading distribution along the chord the strength of the pressure doublet related to the thrust and torque will be given by

$$F_T(r_1,\theta_1,t) = \frac{B}{\pi}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{r_1}{nBb} \sin\frac{nBb}{2r_1}\, e^{inB\Omega\left(t-\frac{\theta_1}{\Omega}-\frac{b}{2\Omega r_1}\right)} \frac{dT}{dr_1} \tag{10}$$

$$F_Q(r_1,\theta_1,t) = \frac{B}{\pi}\sum_{n=-\infty}^{\infty} \frac{r_1}{nBb} \sin\frac{nBb}{2r_1}\, e^{inB\Omega\left(t-\frac{\theta_1}{\Omega}-\frac{b}{2\Omega r_1}\right)} \frac{1}{r_1}\frac{dQ}{dr_1}$$

The strengths of pressure doublets are finally expressed by means of eqs.9 and 10 in terms of the lift coefficients associated with four flow conditions indicated by eq. 8. Then the pressure signals associated with the thrust and torque loading are given by the well known expressions

$$P_T = -\frac{1}{4\pi}\int_0^R\int_0^{2\pi} F_T(r_1,\theta_1,t)\,\frac{\partial}{\partial x}\frac{1}{S}\,dr_1\,d\theta_1$$

$$P_Q = -\frac{1}{4\pi}\int_0^R\int_0^{2\pi} F_Q(r_1,\theta_1,t)\,\frac{\partial}{r_1\partial\theta_1}\frac{1}{S}\,dr_1\,d\theta_1 \tag{11}$$

where r_1, θ_1 = polar coordinates of the doublet singularity on the propeller plane

x, r, θ = cylindrical coordinates of the field point or point of observation

R = propeller radius

S = Descartes' distance between the singularity and field point $= \{x^2+r^2+r_1^2 - 2rr_1 \cos(\theta_1-\theta)\}^{1/2}$

In ref. 3 the pressure signals emanating from the propeller and associated with the thrust and torque loading are determined by means of eq. 11 in conjunction with eqs. 8, 9, 10. The thrust loading pressure which is of interest in the present work is given by (see eq. 20 for the case k = 0 in the above reference)

$$P_T = \frac{e^{inB\Omega t}}{4\pi^2} B \int_0^R C(r_1) \Bigg\{ x \Bigg[\frac{a_o}{2} B'_{3,n} + \sum_{n_1=1}^{\infty} \frac{a_{n_1}}{2} (B'_{3,n+n_1} + B'_{3,n-n_1}) + \sum_{n_1=1}^{\infty} \frac{b_{n_1}}{2} (B''_{3,n+n_1} - B''_{3,n-n_1}) \Bigg] - ix \Bigg[\frac{a_o}{2} B''_{3,n} + \sum_{n_1=1}^{\infty} \frac{a_{n_1}}{2} (B''_{3,n+n_1} - B''_{3,n-n_1}) - \sum_{n_1=1}^{\infty} \frac{b_{n_1}}{2} (B'_{3,n+n_1} - B'_{3,n-n_1}) \Bigg] \Bigg\} dr_1 \tag{12}$$

where

$$B'_{\nu,n} = \frac{\cos nB\theta}{(x^2+r^2+r_1^2)^{\nu/2}} \int_0^{2\pi} \frac{\cos nB\xi}{(1-k_1\cos\xi)^{\nu/2}} d\xi$$

$$B''_{\nu,n} = \frac{\sin nB\theta}{(x^2+r^2+r_1^2)^{\nu/2}} \int_0^{2\pi} \frac{\cos nB\xi}{(1-k_1\cos\xi)^{\nu/2}} d\xi \tag{13}$$

$$k_1 = \frac{2r_1 r}{x^2+r^2+r_1^2} \quad ; \quad C(r_1) = \frac{2r_1}{nBb} \sin \frac{nBb}{2r_1} e^{-i\frac{nBb}{2r_1}}$$

a_{n_1}, b_{n_1} the cosine and sine Fourier coefficients of the loading function (eq. 9) expressed in terms of the harmonic constituents of the wake velocity. Equation 13 can be written in more convenient form as

$$xB_{3,n}' = x \cos nB\theta \int_0^{2\pi} \frac{\cos nB\xi \, d\xi}{\left[(x^2+r^2+r_1^2 - 2rr_1 \cos\xi\right]^{3/2}}$$

$$= -\frac{\partial}{\partial x} \cos nB\theta \int_0^{2\pi} \cos nB\xi \left\{ \sum_{m=0}^{\infty} \epsilon_m \cos m\xi \int_{k'=0}^{\infty} J_m(k'r) J_m(k'r_1) e^{-|x|k'} dk' \right\} d\xi$$

where use is made of the well known Fourier expansion of the inverse of the Descartes' distance 1/S, i.e.:

$$\frac{1}{S} = \sum_{m=0}^{\infty} \epsilon_m \cos m\theta \int_{k'=0}^{\infty} J_m(k'r) J_m(k'r_1) e^{-|x|k'} dk'$$

where $\epsilon_m = 1 \qquad m = 0$

$\epsilon_m = 2 \qquad \text{for } m \neq 0$

Interchanging the order of summation and integration (which is permissible in this case as shown in Appendix 2 of ref. 6) leads to

$$xB_{3,n}' = -2\pi \cos nB\theta \frac{\partial}{\partial x} \int_0^{\infty} J_{nB}(k'r) J_{nB}(k'r_1) e^{-|x|k'} dk'$$

Similarly

$$xB_{3,n}'' = -2\pi \sin nB\theta \frac{\partial}{\partial x} \int_0^{\infty} J_{nB}(k'r) J_{nB}(k'r_1) e^{-|x|k'} dk'$$

Therefore eq. 12 is seen to be made up of terms of the form

$$-\pi a_o \left\{ \begin{matrix} \cos nB\theta \\ \sin nB\theta \end{matrix} \right. \frac{\partial}{\partial x} \int_0^{\infty} J_{nB}(k'r) J_{nB}(k'r_1) e^{-|x|k'} dk'$$

(eq. 14 con't on next pg)

$$-\pi a_{n_1}\begin{cases}\cos(n\pm n_1)B\theta \\ \sin(n\pm n_1)B\theta\end{cases} \frac{\partial}{\partial x}\int_0^\infty J_{(n\pm n_1)B\theta}(k'r)\,J_{(n\pm n_1)B\theta}(k'r_1)\,e^{-|x|k'}dk'$$

$$-\pi b_{n_1}\begin{cases}\cos(n\pm n_1)B\theta \\ \sin(n\pm n_1)B\theta\end{cases} \frac{\partial}{\partial x}\int_0^\infty J_{(n\pm n)B\theta}(k'r)\,J_{(n\pm n_1)B\theta}(k'r_1)\,e^{-|x|k'}dk' \tag{14}$$

where n_1 extends from one to infinity.

However, the indicated θ-integration in eq. 3 fixes the order of harmonics which will contribute to the blade-frequency vibratory force. In fact, the only terms which remain after the θ-integrations are those for $n-n_1=0$ or $n_1=n$ since $n\neq 0$ and $n+n\neq 0$, both being greater than zero.

Therefore, out of an infinite number of possible combinations of the harmonics of the space function with those of the loading function, only the zero order harmonic of the space function with the blade frequency part of the loading contributes to the vibratory force. Hence the blade-frequency vibratory force referred to a coordinate system fixed at the center of the spheroid will be given by

$$F_x = -\frac{B_e^{\,inB\Omega t}}{2}\, a(1-e^2)\int_{-1}^{1}\int_0^R C(r_1)\left[(a_n(r_1)-ib_n(r_1)\right]$$

$$\frac{\partial}{\partial\xi}\int_{k=0}^{\infty} J_0(k'r_1)\,J_0(k'r_0)\,e^{-a|f'+\xi|k'}\,\xi\,dk'\;dr_1 d\xi$$

where

$$r_0' = a(1-e^2)^{1/2}\,(1-\xi^2)^{1/2}$$

$$f' = f/a$$

The ξ - integration by parts leads to

$$F_x = \frac{e^{inB\Omega t}}{2}\, B\, a(1-e^2)\int_0^R C(r_1)\,\left[a_n(r_1)-ib_n(r_1)\right]\left[I_1-I_2\right] \tag{15}$$

where

$$I_1 = \frac{1}{[(f'+a)^2 + r_1^2]^{1/2}} + \frac{1}{[(f'-a)^2 + r_1^2]^{1/2}}$$

$$I_2 = \int_{-1}^{1} \int_{0}^{\infty} J_o(k'r_1)\ J_o(k'r_o)\, e^{-a(f'+\xi)k'}\, dk'\, d\xi$$

or

$$I_2 = \frac{2}{\pi} \int_{-1}^{1} \frac{K(\sigma) \qquad d\xi}{\{a^2(f'+\xi)^2 + \lambda\ (1-\xi^2)^{1/2} + r_1^2\ \}^{1/2}}$$

or

$$I_2 = \frac{1}{\pi\sqrt{r_1}} \int_{-1}^{1} \frac{1}{\sqrt{\lambda(1-\xi^2)}}\ Q_{-1/2}\ (Z)\ d\xi$$

where $K(\sigma)$ = complete elliptic integral of the first kind of modulus

$$\sigma = \frac{2\sqrt{r\lambda(1-\xi^2)^{1/2}}}{\{a^2(f'+\xi)^2 + [\lambda(1-\xi^2)^{1/2}+r_1]^2\}^{1/2}}$$

$$\lambda = a(1-e^2)^{1/2}$$

$Q_{-1/2}$ = Legendre's function of the second kind of -1/2 order

$$Z = \frac{a^2(f'+\xi)^2 + \lambda^2(1-\xi^2) + r_1^2}{2r\lambda\ (1-\xi^2)^{1/2}}$$

In eq. 15 I_1 denotes the bow and stern contribution to the vibratory force and I_2 gives the contribution of the rest of the surface. The second term of I_1 gives the stern contribution, which is the predominant contribution, whereas the first term is associated with the bow contribution.

In the second scheme where the pressure P_T has to be evaluated at $r = 0$, eq. 14 indicates that $P_T \neq 0$ if and only if $n = 0$ or $n \pm n_1 = 0$. The case $n = 0$ is of no interest to us since the blade-frequency force must be evaluated. The case $n + n_1 = 0$ is impossible since n and n_1 vary from 1 to infinity. Therefore the only possibility is $n - n_1 = 0$ or

$n_1 = n$ which reduces eq. 12 to the form

$$P_T = B\,\frac{e^{inB\Omega t}}{4\pi^2}\int_0^R C(r_1)\left\{a_n(r_1)-ib_n(r_1)\right\}\frac{\partial}{\partial x}\int_0^\infty J_o(k'r_1)\,e^{-|x|k'}dk' \tag{16}$$

After substitution of eq. 16 into eq. 7 the longitudinal component of the vibratory force is determined as

$$F_x = -\frac{e^{inB\Omega t}}{E}\int_{x=f-ae}^{f+ae}\int_0^R C(r_1)\left\{a_n(r_1)-ib_n(r_1)\right\}(f-x)$$

$$\frac{\partial}{\partial x}\int_0^\infty J_o(k'r_1)\;e^{-|x|k'}\;dk'\;dr_1\;dx$$

which after the k and x integrations leads to

$$F_x = B\frac{e^{inB\Omega t}}{E}\int_0^R C(r_1)\left\{a_n(r_1)-b_n(r_1)\right\}\left\{\frac{ae}{\sqrt{(f+ae)^2+r_1^2}}+\frac{ae}{\sqrt{(f-ae)^2+r_1^2}}\right.$$

$$\left.+\log\left[\frac{f-ae+\sqrt{(f-ae)^2+r_1^2}}{f+ae+\sqrt{(f+ae)^2+r_1^2}}\right]\right\}dr_1 \tag{17}$$

Comparison of eq. 15 and 17 shows that both are of the same structure and, in fact, the first two terms of eq. 17 can be shown to be identical with the first two terms of eq. 15 for a slender body, with $e \to 1$ and $\frac{ae}{E} \to \frac{a(1-e^2)}{2}$. The logarithmic term is equivalent to the I_2 term, which is small and represents the contribution of the line singularity distribution extended between the focal points.

For both expressions, however, the remaining radial integration must be performed either numerically, since numerical values of the loading function $a_n(r_1)$ are known in terms of the measured wake velocity, or by utilizing the mean-value theorem of the calculus.

The second scheme, however, can be further simplified by assuming that the chordwise and loading functions are evaluated at a suitable radial distance (R_e) and then consecutive integrations with respect to x and r_1 lead to the following form

$$F_x = B\frac{e^{inB\Omega t}}{E}C(R_e)\left\{ a_n(R_e) - ib_n(R_e) \right\} \left\{ Rf \log \frac{R(f+ae) + (f+ae)\sqrt{(f-ae)^2 + R^2}}{R(f-ae) + (f-ae)\sqrt{(f+ae)^2 + R^2}} + R^2 \log \left[\frac{f-ae + \sqrt{(f-ae)^2 + R^2}}{f+ae + \sqrt{(f+ae)^2 + R^2}} \right] \right\} \tag{18}$$

The method of evaluating the Fourier coefficients $a_n(r_1)$ and $b_n(r_1)$ has been indicated sketchily but the necessary information is presented in detail in refs. 3 and 4. With the experience gained from previous calculation, it is suggested that the quasi-steady approach for the loading function be utilized. Making use of the two-dimensional unsteady theory in a stripwise fashion has led to a great discrepancy between the measured and calculated vibratory thrust, whereas the quasi-steady results are close to the experimental. This discrepancy has led to a series of investigations at Davidson Laboratory which apply lifting-surface theory approach to the marine propeller case.

In the quasi-steady flow case, then, the lift per unit of blade radius at each radial distance can be written[4] as

$$L \approx 2\pi\rho c\, K_{gs}\, K_s\, (U^2\alpha^* + UW_c - U\alpha^* W_s - W_s W_c) \tag{19}$$

where

c = semi-chord at that radius

K_{gs}, K_s = correction factors for cascade and finite aspect ratio effects[11]

The remainder of the symbols are as defined previously.

It is suggested that eqs. 18 and 19 be used in the evaluation of the longitudinal component of the vibratory force. Numerical calculations have been obtained by means of eq. 18 to determine the dependence of the longitudinal vibratory force on propeller clearance, as well as on the slenderness ratio of the ellipsoid of revolution, for the same loading conditions and a given propeller radius.

In the limiting case $e \to 1$, when the ellipsoid degenerates to a segment of the x-axis of length 2ae about the origin, it is easily seen that $\lim_{e \to 1} F_x = 0$. When, however, $e \to 0$ and the ellipsoid degenerates to a sphere of radius a, the vibratory force F_x is obtained by three successive applications of L'Hospital's rule as

$$F_x = Be^{inB\Omega t}\, C(R_e) \left\{ a_n(R_e) - ib_n(R_e) \right\} \left\{ \frac{a^3R^2}{(f^2+R^2)^{3/2}} \left(1 + \frac{R^2}{2f^2}\right) \right\} \quad (20)$$

The space factor, $\frac{a^3R^2}{(f^2+R^2)^{3/2}} \left(1 + \frac{R^2}{2f^2}\right)$

compares with $\frac{a^3R^2}{(f^2+R^2)^{3/2}}$ of ref. 10 where the propeller action is represented by a sink disk. The additional term $R^2/2f^2$ is very small in all cases of practical interest, so that the sink disk representation of the propeller action is a good approximation as far as the longitudinal force is concerned. It is interesting to notice, however, that the difference between eq. 20 and that of ref. 10 lies in the form of the loading distribution. The sink representation for the propeller action can be proved exact. In the case treated previously,[10] the entire disk is activated, while in the present case only segments of the disk participate in simulating propeller action. This fact is the actual reason for the observed differences. Thus eq. 20 should be considered more accurate. In the marine propeller case with very wide blades the difference between disk activator and activating sectors should be small.

Calculations indicate the importance of the propeller clearance and the slenderness ratio of the body and, of course, the shape of the afterbody, since the slope of the cross-sectional area curve depends on the slenderness of the body.

In changing the propeller clearance from zero to 1/4 of propeller diameter ($C = .04a$), the vibratory force is reduced by about 50% for slenderness ratio 0.1 to 0.2, which is a region of practical interest. Figure 1 is a chart of the space function (of eqs. 18 and 20) versus slenderness ratio $\frac{b}{a} = \sqrt{1 - e^2}$ for $R = 0.08a$.

CONCLUSION

In the foregoing analysis an expression for the longitudinal component of the vibratory force exerted on an ellipsoid of revolution by the propeller action has been developed in closed form in terms of elementary functions, and the relative importance of the various parameters has been revealed. It is indicated that the vibratory force strongly depends on the axial propeller clearance as well as on the slenderness of the ellipsoid of revolution and therefore on the slope of the cross-sectional area at the afterbody section. In fact, in the region of practical interest, for slenderness ratio of 0.1 to 0.2, changing the propeller clearance from zero to 1/4 propeller diameter reduces the longitudinal vibratory force to approximately half. In this range, also, the force decreases rapidly with increasing fineness.

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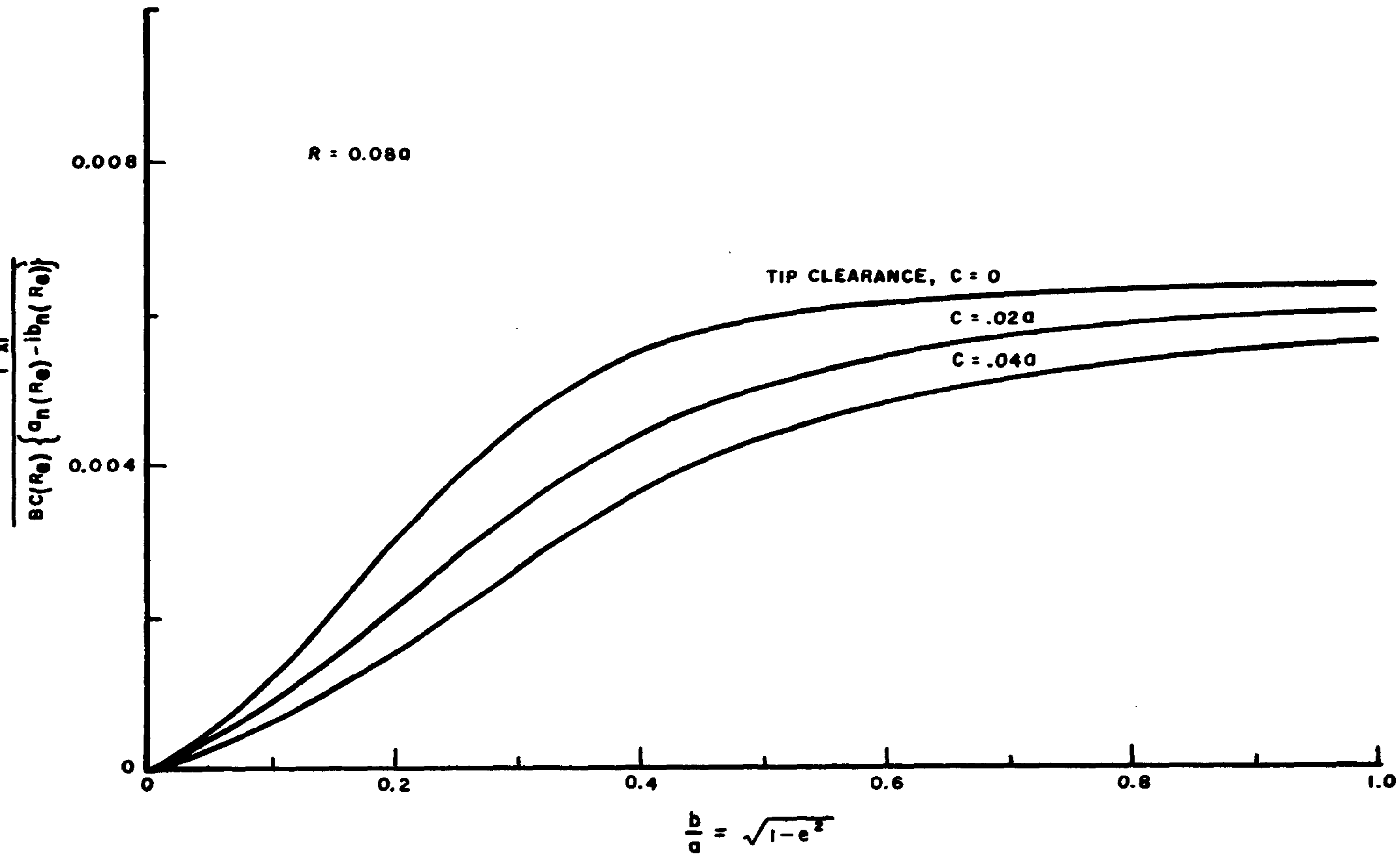

FIGURE I. VARIATION OF SPACE FUNCTION WITH SLENDERNESS OF SPHEROID AT VARIOUS PROPELLER CLEARANCES

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